Hoofdstuk 12: Differentiaalrekening

12.1 De productregel

Opgave 1:

a. $p(x) = f(x) \cdot g(x) = x^2 \cdot (3x - 7) = 3x^3 - 7x^2$

$p'(x) = 9x^2 - 14x$

$f'(x) = 2x$

$g'(x) = 14x$

b. nee want $f'(x) \cdot g'(x) = 2x \cdot 14x = 28x^2$

c. $p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

$= 2x(3x - 7) + x^2 \cdot 3$

$= 6x^2 - 14x + 3x^2$

$= 9x^2 - 14x$

Opgave 2:

a. $f(x) = x^2(2x - 1)$

$f'(x) = 2x(2x - 1) + x^2 \cdot 2$

$= 2x(2x - 1) + 2x^2$

b. $g(x) = 2x^3(x^2 - 3)$

$g'(x) = 6x^2(x^2 - 3) + 2x^3 \cdot 2x$

$= 6x^2(x^2 - 3) + 4x^4$

c. $h(x) = (x^2 - 1)(x^2 + 3)$

$h'(x) = 2x(x^2 + 3) + (x^2 - 1) \cdot 2x$

d. $j(x) = (2x^3 + 1)(3x^2 - 1)$

$j'(x) = 6x^2(3x^2 - 1) + (2x^3 + 1) \cdot 6x$

Opgave 3:

a. $f(x) = (2 - 3x^2)(2 + 7x)$

$f'(x) = -6x(2 + 7x) + (2 - 3x^2) \cdot 7$

b. $g(x) = (2x - 5)^2 = (2x - 5)(2x - 5)$

$g'(x) = 2(2x - 5) + (2x - 5) \cdot 2$

$= 4(2x - 5)$

c. $h(x) = (x^2 - 3x)(x^3 + x^2 + x)$

$h'(x) = (2x - 3)(x^3 + x^2 + x) + (x^2 - 3x)(3x^2 + 2x + 1)$

d. $j(x) = (3x^2 - 4)^2 = (3x^2 - 4)(3x^2 - 4)$

$j'(x) = 6x(3x^2 - 4) + (3x^2 - 4) \cdot 6x$

$= 12x(3x^2 - 4)$

Opgave 4:

$g(x) = c \cdot f(x)$

$g'(x) = 0 \cdot f(x) + c \cdot f'(x) = c \cdot f'(x)$

Opgave 5:

a. $f(x) = (4x^2 - 1)(3x + 2)$

$\quad = 12x^3 + 8x^2 - 3x - 2$

$f'(x) = 36x^2 + 16x - 3$

$f'(-1) = 17$

$f(-1) = -3$

$y = 17x + b$ door $(-1,-3)$

$-3 = -17 + b$

$b = 14$

$y = 17x + 14$

b. $B(0,-2)$

$f'(0) = -3$

$y = -3x + b$ door $(0,-2)$

$b = -2$

Opgave 6:

a. $f(x) = (x^2 + 1)(x^2 - 4) = 0$

$x^2 = -1 \quad \vee \quad x^2 = 4$

$x = 2 \quad \vee \quad x = -2$

$f(x) = (x^2 + 1)(x^2 - 4)$

$\quad = x^4 - 3x^2 - 4$

$f'(x) = 4x^3 - 6x$

$f'(2) = 20$	$f'(-2) = -20$
$y = 20x + b$ door $(2,0)$	$y = -20x + b$ door $(-2,0)$
$0 = 40 + b$	$0 = 40 + b$
$b = -40$	$b = -40$
$y = 20x - 40$	$y = 20x - 40$

b. $f'(1) = -2 \neq 0$ dus geen extreme waarde

c. $f'(\sqrt{1\tfrac{1}{2}}) = 4 \cdot \sqrt{1\tfrac{1}{2}}^3 - 6\sqrt{1\tfrac{1}{2}} = 4 \cdot 1\tfrac{1}{2}\sqrt{1\tfrac{1}{2}} - 6\sqrt{1\tfrac{1}{2}} = 6\sqrt{1\tfrac{1}{2}} - 6\sqrt{1\tfrac{1}{2}} = 0$

Opgave 7:

a. $P(p,0)$ dus $S(p, -p^2 + 6p)$

$y = -x^2 + 6x = 0$

$-x(x - 6) = 0$

$x = 0 \quad \vee \quad x = 6$

$Q = 6 - p$

$PQ = 6 - p - p = 6 - 2p$

$PS = -p^2 + 6p$

$A = (6 - 2p)(-p^2 + 6p)$

b. $A = (6 - 2p)(-p^2 + 6p)$

$\quad = -6p^2 + 36p + 2p^3 - 12p^2$

$\quad = 2p^3 - 18p^2 + 36p$

$A' = 6p^2 - 36p + 36$

c. $6p^2 - 36p + 36 = 0$

$p^2 - 6p + 6 = 0$

$p = \frac{6 \pm \sqrt{12}}{2} = \frac{6 \pm 2\sqrt{3}}{2} = 3 \pm \sqrt{3}$

$p < 3$ dus $p = 3 - \sqrt{3}$

<u>Opgave 8:</u>

a. $f(x) = (\tfrac{1}{2}x^3 - 4)^2 - 5$

$= \tfrac{1}{4}x^6 - 4x^3 + 16 - 5$

$= \tfrac{1}{4}x^6 - 4x^3 + 11$

$f'(x) = 1\tfrac{1}{2}x^5 - 12x^2$

$f'(1) = -10\tfrac{1}{2}$

$y = f(1) = 7\tfrac{1}{4}$

$y = -10\tfrac{1}{2}x + b$ door $(1, 7\tfrac{1}{4})$

$7\tfrac{1}{4} = -10\tfrac{1}{2} + b$

$17\tfrac{3}{4} = b$

$y = -10\tfrac{1}{2}x + 17\tfrac{3}{4}$

b. $B(0,11)$

$f'(0) = 0$

$y = 0x + b$ door $(0,11)$

$b = 11$

$y = 11$

c. $f'(x) = 1\tfrac{1}{2}x^5 - 12x^2 = 0$

$1\tfrac{1}{2}x^2(x^3 - 8) = 0$

$x = 0 \quad \vee \quad x^3 = 8$

$x = 0 \quad \vee \quad x = 2$

(2,–5)

<u>Opgave 9:</u>

a. $AC = 4 - p$

$B(p, p^2 - 2p + 3)$

$Opp = \tfrac{1}{2}(4 - p)(p^2 - 2p + 3)$

$= (2 - \tfrac{1}{2}p)(p^2 - 2p + 3)$

b. $Opp = (2 - \tfrac{1}{2}p)(p^2 - 2p + 3)$

$= 2p^2 - 4p + 6 - \tfrac{1}{2}p^3 + p^2 - 1\tfrac{1}{2}p$

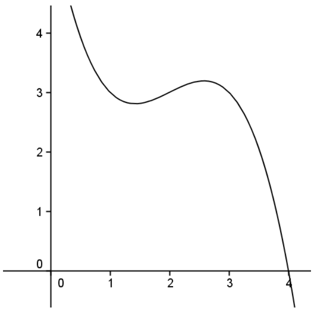
$= -\tfrac{1}{2}p^3 + 3p^2 - 5\tfrac{1}{2}p + 6$

$Opp' = -1\tfrac{1}{2}p^2 + 6p - 5\tfrac{1}{2} = 0$

$p = \frac{-6 \pm \sqrt{3}}{-3}$

$p = 1{,}42 \quad \vee \quad p = 2{,}58$

dus $p = 2{,}58$



12.2 De afgeleide van machtsfuncties

Opgave 10:

a. $\frac{1}{x^3} = x^{-3}$

$\frac{5}{x^4} = 5x^{-4}$

$\frac{1}{3x^2} = \frac{1}{3}x^{-2}$

b. $x^{-4} = \frac{1}{x^4}$

$3x^{-2} = \frac{3}{x^2}$

$\frac{1}{7}x^{-6} = \frac{1}{7x^6}$

Opgave 11:

a. $h(x) = f(x) \cdot g(x) = x^2 \cdot x^{-2}$

$h'(x) = 2x \cdot x^{-2} + x^2 \cdot \left[x^{-2}\right]'$

$= 2x^{-1} + x^2 \cdot \left[x^{-2}\right]'$

b. $h(x) = x^2 \cdot x^{-2} = x^{2+-2} = x^0 = 1$

$h'(x) = 0$

c. $2x^{-1} + x^2 \cdot \left[x^{-2}\right]' = 0$

$x^2 \cdot \left[x^{-2}\right]' = -2x^{-1}$

$\left[x^{-2}\right]' = \frac{-2x^{-1}}{x^2}$

d. vul $n = -2$ in dan krijg je opgave c

Opgave 12:

a. $f(x) = \frac{5}{x^3} = 5x^{-3}$

$f'(x) = -15x^{-4} = \frac{-15}{x^4}$

b. $g(x) = \frac{1}{5x^3} = \frac{1}{5}x^{-3}$

$g'(x) = \frac{-3}{5}x^{-4} = \frac{-3}{5x^4}$

c. $h(x) = 5x^2 - \frac{5}{x^2} = 5x^2 - 5x^{-2}$

$h'(x) = 10x + 10x^{-3} = 10x + \frac{10}{x^3}$

Opgave 13:

a. $f(x) = \frac{x^4 - 5}{x^3} = \frac{x^4}{x^3} - \frac{5}{x^3} = x - 5x^{-3}$

$f'(x) = 1 + 15x^{-4} = 1 + \frac{15}{x^4}$

b. $g(x) = \frac{2x^2 - 3}{x^3} = \frac{2x^2}{x^3} - \frac{3}{x^3} = 2x^{-1} - 3x^{-3}$

$g'(x) = -2x^{-2} + 9x^{-4} = \frac{-2}{x^2} + \frac{9}{x^4}$

c. $h(x) = \frac{x+2}{3x} = \frac{x}{3x} + \frac{2}{3x} = \frac{1}{3} + \frac{2}{3}x^{-1}$

$h'(x) = -\frac{2}{3}x^{-2} = \frac{-2}{3x^2}$

Opgave 14:

a. $f(x) = \frac{2x-1}{3x^2} = \frac{2x}{3x^2} - \frac{1}{3x^2} = \frac{2}{3}x^{-1} - \frac{1}{3}x^{-2}$

$f'(x) = -\frac{2}{3}x^{-2} + \frac{2}{3}x^{-3} = \frac{-2}{3x^2} + \frac{2}{3x^3}$

b. $g(x) = 6 - \frac{x^2 - 1}{x} = 6 - \frac{x^2}{x} + \frac{1}{x} = 6 - x + x^{-1}$

$g'(x) = -1 - x^{-2} = -1 - \frac{1}{x^2}$

c. $h(x) = \frac{5}{2x^2} - \frac{2x^2}{5} = \frac{5}{2}x^{-2} - \frac{2}{5}x^2$

$h'(x) = -5x^{-3} - \frac{4}{5}x = \frac{-5}{x^3} - \frac{4}{5}x$

Opgave 15:

a. kruiselings vermenigvuldigen geeft $5 \cdot 2 = 10 \cdot 1$

b. $\frac{5}{10} = \frac{x}{2}$

kruiselings vermenigvuldigen geeft: $10 \cdot x = 5 \cdot 2$

c. $\frac{5}{6} = \frac{3}{x}$

$5x = 18$

$x = 3{,}6$

Opgave 16:

a. $\frac{4}{x^2} = \frac{1}{9}$

$x^2 = 36$

$x = 6 \quad \vee \quad x = -6$

b. $\frac{x-3}{x} = \frac{2}{7}$

$7(x-3) = 2x$

$7x - 21 = 2x$

$5x = 21$
$x = 4{,}2$

c. $\frac{x}{x-4} = \frac{3}{8}$
$8x = 3(x-4)$
$8x = 3x - 12$
$5x = -12$
$x = -2{,}4$

d. $3 - \frac{6}{x^2} = 1\tfrac{1}{2}$
$-\frac{6}{x^2} = -1\tfrac{1}{2}$
$-1\tfrac{1}{2}x^2 = -6$
$x^2 = 4$
$x = 2 \quad \vee \quad x = -2$

Opgave 17:

a. $f(x) = \frac{2x+3}{x} = 0$
$2x + 3 = 0$
$2x = -3$
$x = -1\tfrac{1}{2}$
$f(x) = \frac{2x+3}{x} = \frac{2x}{x} + \frac{3}{x} = 2 + 3x^{-1}$
$f'(x) = -3x^{-2} = \frac{-3}{x^2}$
$f'(-1\tfrac{1}{2}) = -\tfrac{4}{3}$
$y = -\tfrac{4}{3}x + b$ door $(-1\tfrac{1}{2}, 0)$
$0 = 2 + b$
$b = -2$
$y = -\tfrac{4}{3}x - 2$

b. $f'(x) = \frac{-3}{x^2} = -\tfrac{3}{4}$
$x^2 = 4$
$x = 2 \quad \vee \quad x = -2$
$(2, 3\tfrac{1}{2})$ en $(-2, \tfrac{1}{2})$

Opgave 18:

a. $\frac{x+1}{x-1} = \frac{x+3}{x}$
$x(x+1) = (x+3)(x-1)$
$x^2 + x = x^2 + 2x - 3$
$-x = -3$
$x = 3$

b. $\dfrac{x}{x+2}=\dfrac{3}{x-2}$

$x(x-2)=3(x+2)$

$x^2-2x=3x+6$

$x^2-5x-6=0$

$(x-6)(x+1)=0$

$x=6 \quad \vee \quad x=-1$

c. $\dfrac{x}{x+4}=\dfrac{1}{2x-1}$

$x(2x-1)=x+4$

$2x^2-x=x+4$

$2x^2-2x-4=0$

$x^2-x-1=0$

$(x-2)(x+1)=0$

$x=2 \quad \vee \quad x=-1$

d. $\dfrac{4x}{x+2}+3=x$

$\dfrac{4x}{x+2}=x-3$

$(x+2)(x-3)=4x$

$x^2-x-6=4x$

$x^2-5x-6=0$

$(x-6)(x+1)=0$

$x=6 \quad \vee \quad x=-1$

Opgave 19:

a. $f(x)=\dfrac{x^2+4}{x}=\dfrac{x^2}{x}+\dfrac{4}{x}=x+4x^{-1}$

$f'(x)=1-4x^{-2}=1-\dfrac{4}{x^2}$

$f'(3)=\frac{5}{9}$

$x_A=3 \quad y_A=f(3)=4\frac{1}{3}$

$y=\frac{5}{9}x+b$ door $(3,4\frac{1}{3})$

$4\frac{1}{3}=\frac{5}{3}+b$

$2\frac{2}{3}=b$

$y=\frac{5}{9}x+2\frac{2}{3}$

b. $f'(x)=1-\dfrac{4}{x^2}=-3$

$-\dfrac{4}{x^2}=-4$

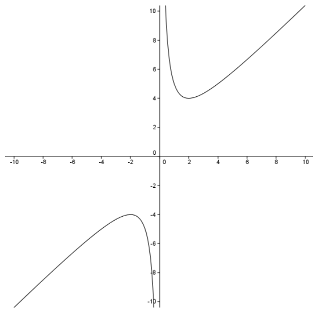
$-4x^2=-4$

$x^2=1$

$x=1 \quad \vee \quad x=-1$

$(1,5)$ en $(-1,-5)$

c. $f'(x) = 1 - \dfrac{4}{x^2} = 0$



$-\dfrac{4}{x^2} = -1$

$-x^2 = -4$

$x^2 = 4$

$x = 2 \quad \vee \quad x = -2$

max $f(-2) = -4$

min $f(2) = 4$

d. $f'(x) = 1 - \dfrac{4}{x^2} = 2$

$-\dfrac{4}{x^2} = 1$

$x^2 = -4$

geen oplossingen

Opgave 20:

a. $y = x^{\frac{1}{2}} \cdot x^{\frac{1}{2}}$

$y' = \left[x^{\frac{1}{2}}\right]' \cdot x^{\frac{1}{2}} + x^{\frac{1}{2}} \cdot \left[x^{\frac{1}{2}}\right]' = 2x^{\frac{1}{2}} \cdot \left[x^{\frac{1}{2}}\right]'$

als $y = x$ dan $y' = 1$

$2x^{\frac{1}{2}} \cdot \left[x^{\frac{1}{2}}\right]' = 1$

b. $\left[x^{\frac{1}{2}}\right]' = \dfrac{1}{2x^{\frac{1}{2}}} = \frac{1}{2}x^{-\frac{1}{2}}$

Opgave 21:

a. $f(x) = x + \sqrt{x} = x + x^{\frac{1}{2}}$

$f'(x) = 1 + \frac{1}{2}x^{-\frac{1}{2}} = 1 + \dfrac{1}{2x^{\frac{1}{2}}} = 1 + \dfrac{1}{2\sqrt{x}}$

b. $g(x) = x \cdot \sqrt[3]{x} = x \cdot x^{\frac{1}{3}} = x^{1\frac{1}{3}}$

$g'(x) = 1\frac{1}{3}x^{\frac{1}{3}} = 1\frac{1}{3} \cdot \sqrt[3]{x}$

c. $h(x) = \dfrac{1}{\sqrt{x}} = \dfrac{1}{x^{\frac{1}{2}}} = x^{-\frac{1}{2}}$

$h'(x) = -\frac{1}{2}x^{-1\frac{1}{2}} = \dfrac{-1}{2x^{1\frac{1}{2}}} = \dfrac{-1}{2x\sqrt{x}}$

d. $j(x) = x^3 \cdot \sqrt[5]{x^3} = x^3 \cdot x^{\frac{3}{5}} = x^{3\frac{3}{5}}$

$j'(x) = 3\frac{3}{5}x^{2\frac{3}{5}} = 3\frac{3}{5}x^2 \cdot x^{\frac{3}{5}} = 3\frac{3}{5}x^2 \cdot \sqrt[5]{x^3}$

e. $k(x) = x^2 \cdot \sqrt[4]{x} = x^2 \cdot x^{\frac{1}{4}} = x^{2\frac{1}{4}}$

$k'(x) = 2\frac{1}{4}x^{1\frac{1}{4}} = 2\frac{1}{4}x \cdot x^{\frac{1}{4}} = 2\frac{1}{4}x \cdot \sqrt[4]{x}$

f. $l(x) = (x^2+1)(1+\sqrt{x}) = x^2 + x^2\sqrt{x} + 1 + \sqrt{x} = x^2 + x^{2\frac{1}{2}} + 1 + x^{\frac{1}{2}}$

$l'(x) = 2x + 2\tfrac{1}{2}x^{1\frac{1}{2}} + \tfrac{1}{2}x^{-\frac{1}{2}} = 2x + 2\tfrac{1}{2}x\sqrt{x} + \dfrac{1}{2x^{\frac{1}{2}}} = 2x + 2\tfrac{1}{2}x\sqrt{x} + \dfrac{1}{2\sqrt{x}}$

Opgave 22:

a. $f(x) = \dfrac{4x^2+1}{x\sqrt{x}} = \dfrac{4x^2}{x^{1\frac{1}{2}}} + \dfrac{1}{x^{1\frac{1}{2}}} = 4x^{\frac{1}{2}} + x^{-1\frac{1}{2}}$

$f'(x) = 2x^{-\frac{1}{2}} - 1\tfrac{1}{2}x^{-2\frac{1}{2}} = \dfrac{2}{x^{\frac{1}{2}}} - \dfrac{3}{2x^{2\frac{1}{2}}} = \dfrac{2}{\sqrt{x}} - \dfrac{3}{2x^2\sqrt{x}}$

b. $g(x) = \dfrac{x-4}{\sqrt[3]{x}} = \dfrac{x}{x^{\frac{1}{3}}} - \dfrac{4}{x^{\frac{1}{3}}} = x^{\frac{2}{3}} - 4x^{-\frac{1}{3}}$

$g'(x) = \tfrac{2}{3}x^{-\frac{1}{3}} + \tfrac{4}{3}x^{-1\frac{1}{3}} = \dfrac{2}{3x^{\frac{1}{3}}} + \dfrac{4}{3x^{1\frac{1}{3}}} = \dfrac{2}{3\cdot\sqrt[3]{x}} + \dfrac{4}{3x\sqrt[3]{x}}$

c. $h(x) = (x\sqrt{x}-3)^2 = x^3 - 6x\sqrt{x} + 9 = x^3 - 6x^{1\frac{1}{2}} + 9$

$h'(x) = 3x^2 - 9x^{\frac{1}{2}} = 3x^2 - 9\sqrt{x}$

Opgave 23:

$x_A = \tfrac{1}{8} \quad y_A = \sqrt[3]{\tfrac{1}{64}} = \tfrac{1}{4}$

$x_B = 8 \quad y_B = \sqrt[3]{64} = 4$

$f(x) = \sqrt[3]{x^2} = x^{\frac{2}{3}}$

$f'(x) = \tfrac{2}{3}x^{-\frac{1}{3}} = \dfrac{2}{3x^{\frac{1}{3}}} = \dfrac{2}{3\cdot\sqrt[3]{x}}$

$f'(\tfrac{1}{8}) = \tfrac{4}{3}$

$y = \tfrac{4}{3}x + b$ door $(\tfrac{1}{8}, \tfrac{1}{4})$

$\tfrac{1}{4} = \tfrac{1}{6} + b$

$\tfrac{1}{12} = b$

$y = \tfrac{4}{3}x + \tfrac{1}{12}$

$f'(8) = \tfrac{1}{3}$

$y = \tfrac{1}{3}x + b$ door $(8,4)$

$4 = 2\tfrac{2}{3} + b$

$1\tfrac{1}{3} = b$

$y = \tfrac{1}{3}x + 1\tfrac{1}{3}$

$\tfrac{4}{3}x + \tfrac{1}{12} = \tfrac{1}{3}x + 1\tfrac{1}{3}$

$x = 1\tfrac{1}{4}$

$C(1\tfrac{1}{4}, 1\tfrac{3}{4})$

Opgave 24:

a. $f(x) = x\sqrt{x} - 3x = x^{1\frac{1}{2}} - 3x$

$f'(x) = 1\tfrac{1}{2}x^{\frac{1}{2}} - 3 = 1\tfrac{1}{2}\sqrt{x} - 3 = 0$

$1\tfrac{1}{2}\sqrt{x} = 3$

$\sqrt{x} = 2$

$x = 4$

min $f(4) = -4$

b. $f'(0) = -3$

$y = -3x$

c. $f'(x) = 1\tfrac{1}{2}\sqrt{x} - 3 = 3$

$1\frac{1}{2}\sqrt{x} = 6$
$\sqrt{x} = 4$
$x = 16$
$y = 16$
$y = 3x + b$ door $(16,16)$
$16 = 48 + b$
$-32 = b$
$y = 3x - 32$

<u>Opgave 25:</u>

a. $y_1 = 10x\sqrt{x}$
$\left[\frac{dy}{dx}\right]_{x=1} = 15$

b. $s = 10t\sqrt{t} = 10t^{1\frac{1}{2}}$
$s' = 15t^{\frac{1}{2}} = 15\sqrt{t}$
$s'(8) = 15\sqrt{8}$

c. $108\ {}^{km}\!/_{uur} = 30\ {}^{m}\!/_{s}$
$s' = 15\sqrt{t} = 30$
$t = 4$

d. $s'(9) = 45\ {}^{m}\!/_{s}$
$s(9) = 270$
$270 + 51 \cdot 45 = 2565\ \text{m}$

<u>Opgave 26:</u>

a. $f(x) = \dfrac{x^3+2}{\sqrt{x}} = \dfrac{x^3}{\sqrt{x}} + \dfrac{2}{\sqrt{x}} = x^{2\frac{1}{2}} + \dfrac{2}{x^{\frac{1}{2}}} = x^{2\frac{1}{2}} + 2x^{-\frac{1}{2}}$

$f'(x) = 2\frac{1}{2}x^{1\frac{1}{2}} - x^{-1\frac{1}{2}} = 2\frac{1}{2}x\sqrt{x} - \dfrac{1}{x^{1\frac{1}{2}}} = 2\frac{1}{2}x\sqrt{x} - \dfrac{1}{x\sqrt{x}}$

$x_A = 1$
$y_A = 3$
$f'(1) = 1\frac{1}{2}$
$y = 1\frac{1}{2}x + b$ door $(1,3)$
$3 = 1\frac{1}{2} + b$
$b = 1\frac{1}{2}$
$y = 1\frac{1}{2}x + 1\frac{1}{2}$

b. $f'(x) = 2\frac{1}{2}x\sqrt{x} - \dfrac{1}{x\sqrt{x}} = 0$

$2\frac{1}{2}x\sqrt{x} = \dfrac{1}{x\sqrt{x}}$

$2\frac{1}{2}x^3 = 1$
$x^3 = \frac{2}{5}$
$x = \sqrt[3]{\frac{2}{5}}$ dus $p = \frac{2}{5}$

c. $y = \dfrac{\frac{2}{5} + 2}{\sqrt{\sqrt[3]{\frac{2}{5}}}} = \dfrac{2\frac{2}{5}}{\sqrt{\left(\frac{2}{5}\right)^{\frac{1}{3}}}} = \dfrac{2\frac{2}{5}}{\left(\left(\frac{2}{5}\right)^{\frac{1}{3}}\right)^{\frac{1}{2}}} = \dfrac{2\frac{2}{5}}{\left(\frac{2}{5}\right)^{\frac{1}{6}}} = \dfrac{2\frac{2}{5}}{\sqrt[6]{\frac{2}{5}}}$

$a = 2\frac{2}{5} \quad b = 6 \quad c = \frac{2}{5}$

12.3 De kettingregel

Opgave 27:

a. $f(x)=(x^2-5x)^2=x^4-10x^3+25x^2$

$f'(x)=4x^3-30x^2+50x$

b. $2(x^2-5x)\cdot\left[x^2-5x\right]=2(x^2-5x)(2x-5)$

$=(x^2-5x)(4x-10)$

$=4x^3-10x^2-20x^2+50x$

$=4x^3-30x^2+50x=f'(x)$

Opgave 28:

De tabellen zijn gelijk

Opgave 29:

a. $f(x)=\sqrt{x^2+4}=\sqrt{u}=u^{\frac{1}{2}}$ met $u=x^2+4$

dus $u'=2x$

$f'(x)=\frac{1}{2}u^{-\frac{1}{2}}\cdot u'=\frac{1}{2u^{\frac{1}{2}}}\cdot u'=\frac{1}{2\sqrt{u}}\cdot u'=\frac{1}{2\sqrt{x^2+4}}\cdot 2x=\frac{x}{\sqrt{x^2+4}}$

b. $g(x)=(2x^4+x^2)^3=u^3$ met $u=2x^4+x^2$

dus $u'=8x^3+2x$

$g'(x)=3u^2\cdot u'=3(2x^4+x^2)^2\cdot(8x^3+2x)$

c. $h(x)=\sqrt[3]{x^3+3x}=\sqrt[3]{u}=u^{\frac{1}{3}}$ met $u=x^3+3x$

dus $u'=3x^2+3$

$h'(x)=\frac{1}{3}u^{-\frac{2}{3}}\cdot u'=\frac{1}{3u^{\frac{2}{3}}}\cdot u'=\frac{u'}{3\sqrt[3]{u^2}}=\frac{3x^2+3}{3\sqrt[3]{(x^3+3x)^2}}$

d. $j(x)=(2x+1)^{-2}=u^{-2}$ met $u=2x+1$

dus $u'=2$

$j'(x)=2u^{-3}\cdot u'=\frac{-2}{u^3}\cdot u'=\frac{-2}{(2x+1)^3}\cdot 2=\frac{-4}{(2x+1)^3}$

Opgave 30:

a. $f(x)=\frac{1}{(3x+1)^2}=\frac{1}{u^2}=u^{-2}$ met $u=3x+1$

dus $u'=3$

$f'(x)=-2u^{-3}\cdot u'=\frac{-2}{u^3}\cdot u'=\frac{-2}{(3x+1)^3}\cdot 3=\frac{-6}{(3x+1)^3}$

b. $g(x)=\frac{1}{\sqrt{4x-1}}=\frac{1}{\sqrt{u}}=\frac{1}{u^{\frac{1}{2}}}=u^{-\frac{1}{2}}$ met $u=4x-1$

dus $u'=4$

$g'(x)=-\frac{1}{2}u^{-1\frac{1}{2}}\cdot u'=\frac{-1}{2u^{1\frac{1}{2}}}\cdot u'=\frac{-1}{2u\sqrt{u}}\cdot u'=\frac{-1}{2(4x-1)\sqrt{4x-1}}\cdot 4=\frac{-2}{(4x-1)\sqrt{4x-1}}$

c. $h(x)=(x^2+4)\sqrt{x^2+4}=u\sqrt{u}=u^{1\frac{1}{2}}$ met $u=x^2+4$

dus $u'=2x$

$h'(x)=1\tfrac{1}{2}u^{\frac{1}{2}}\cdot u'=1\tfrac{1}{2}\sqrt{u}\cdot u'=1\tfrac{1}{2}\sqrt{x^2+4}\cdot 2x=3x\sqrt{x^2+4}$

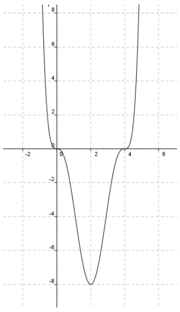
d. $j(x)=\dfrac{x^2+4}{\sqrt{x^2+4}}=\dfrac{u}{\sqrt{u}}=\sqrt{u}=u^{\frac{1}{2}}$ met $u=x^2+4$

dus $u'=2x$

$j'(x)=\tfrac{1}{2}u^{-\frac{1}{2}}\cdot u'=\dfrac{1}{2\sqrt{x^2+4}}\cdot 2x=\dfrac{x}{\sqrt{x^2+4}}$

Opgave 31:

a.



b. $f(x)=(\tfrac{1}{2}x^2-2x)^3=u^3$ met $u=\tfrac{1}{2}x^2-2x$

dus $u'=x-2$

$f'(x)=3u^2\cdot u'=3(\tfrac{1}{2}x^2-2x)^2\cdot(x-2)=0$

$\tfrac{1}{2}x^2-2x=0 \quad \vee \quad x-2=0$

$\tfrac{1}{2}x(x-4)=0 \quad \vee \quad x=2$

$x=0 \quad \vee \quad x=4 \quad \vee \quad x=2$

c. $y_A=f(6)=216$

$f'(6)=432$

$y=432x+b$ door $(6,216)$

$216=2592+b$

$-2376=b$

$y=432x-2376$

Opgave 32:

a. $f(x)=\sqrt{x^2+9}-x^2+5x$

$f'(x)=\dfrac{1}{2\sqrt{x^2+9}}\cdot 2x-2x+5=\dfrac{x}{\sqrt{x^2+9}}-2x+5$

$y_A=f(4)=9$

$f'(4)=-2{,}2$

$y=-2{,}2x+b$ door $(4,9)$

$9=-8{,}8+b$

$17{,}8=b$

$y=-2{,}2x+17{,}8$

b. $f'(3)=-0{,}29\neq 0$ dus geen extreme waarde

Opgave 33:
De productregel vanwege het product van x en $\sqrt{2x+1}$, de kettingregel vanwege $\sqrt{2x+1}$.

Opgave 34:
a. $f(x) = x\sqrt{3x+1}$

$$f'(x) = 1\cdot\sqrt{3x+1} + x\cdot\frac{1}{2\sqrt{3x+1}}\cdot 3 = \sqrt{3x+1} + \frac{3x}{2\sqrt{3x+1}}$$

b. $g(x) = x(3x+1)^3$

$$g'(x) = 1\cdot(3x+1)^3 + x\cdot 3(3x+1)^2\cdot 3 = (3x+1)^3 + 9x(3x+1)^2$$

Opgave 35:
a. $8-2x \geq 0$
$-2x \geq -8$
$x \leq 4$
$D_f = \langle\leftarrow,4]$

b. $f'(x) = 1\cdot\sqrt{8-2x} + x\cdot\frac{1}{2\sqrt{8-2x}}\cdot -2 = \sqrt{8-2x} - \frac{x}{\sqrt{8-2x}}$

c. $\sqrt{8-2x} = \frac{x}{\sqrt{8-2x}}$ kruiselings vermenigvuldigen geeft:
$8-2x = x$

d. $-3x = -8$
$x = \frac{8}{3}$
$y = \frac{8}{3}\sqrt{\frac{8}{3}}$

e. $B_f = \langle\leftarrow,\frac{8}{3}\sqrt{\frac{8}{3}}]$

Opgave 36:
a. $2x+6 \geq 0$
$2x \geq -6$
$x \geq -3$

b. $f'(x) = 1\cdot\sqrt{2x+6} + x\cdot\frac{1}{2\sqrt{2x+6}}\cdot 2$

$$= \sqrt{2x+6} + \frac{x}{\sqrt{2x+6}} = 0$$

$\frac{x}{\sqrt{2x+6}} = -\sqrt{2x+6}$
$x = -(2x+6)$
$x = -2x-6$
$3x = -6$
$x = -2$
$y = -2\sqrt{2}$

c. $B_f = [-2\sqrt{2},\rightarrow\rangle$

Opgave 37:

a. $A(0,-3)$

$f'(x) = 2\sqrt{9-2x} + 2x \cdot \frac{1}{2\sqrt{9-2x}} \cdot -2 = 2\sqrt{9-2x} - \frac{2x}{\sqrt{9-2x}}$

$f'(0) = 6$

$y = 6x + b$ door $(0,-3)$

$y = 6x - 3$

b. $2\sqrt{9-2x} - \frac{2x}{\sqrt{9-2x}} = 0$

$2\sqrt{9-2x} = \frac{2x}{\sqrt{9-2x}}$

$2(9-2x) = 2x$

$18 - 4x = 2x$

$-6x = -18$

$x = 3$

max $f(3) = 6\sqrt{3} - 3$

c. $9 - 2x \geq 0$

$-2x \geq -9$

$x \leq 4\tfrac{1}{2}$

$D_f = \langle \leftarrow, 4\tfrac{1}{2}]$

$B_f = \langle \leftarrow, 6\sqrt{3} - 3]$

12.4 Goniometrische vergelijkingen

Opgave 38:

a. $AD = \frac{1}{2} AB = 1$

$CD = \sqrt{AC^2 - AD^2} = \sqrt{2^2 - 1^2} = \sqrt{3}$

b. $\sin 60° = \frac{CD}{AC} = \frac{\sqrt{3}}{2} = \frac{1}{2}\sqrt{3}$

$\cos 60° = \frac{AD}{AC} = \frac{1}{2}$

c. $\sin 30° = \frac{AD}{AC} = \frac{1}{2}$

$\cos 30° = \frac{CD}{AC} = \frac{\sqrt{3}}{2} = \frac{1}{2}\sqrt{3}$

d. $AC = \sqrt{AB^2 + BC^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\sin 45° = \frac{BC}{AC} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2}$

$\cos 45° = \frac{AB}{AC} = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}$

Opgave 39:

a. $\sin \frac{3}{4}\pi = \frac{1}{2}\sqrt{2}$

b. $\cos \frac{7}{6}\pi = -\frac{1}{2}\sqrt{3}$

c. $\sin 1\frac{1}{3}\pi = -\frac{1}{2}\sqrt{3}$

d. $\cos \frac{5}{3}\pi = \frac{1}{2}$

e. $\cos 1\frac{1}{3}\pi = -\frac{1}{2}$

f. $\sin(-\frac{1}{4}\pi) = -\frac{1}{2}\sqrt{2}$

Opgave 40:

a. $\sin\alpha = \frac{1}{2}\sqrt{3}$

$\alpha = \frac{1}{3}\pi \quad \vee \quad \alpha = \frac{2}{3}\pi$

b. $\cos\alpha = -\frac{1}{2}$

$\alpha = \frac{2}{3}\pi \quad \vee \quad \alpha = 1\frac{1}{3}\pi$

c. $\sin\alpha = -\frac{1}{2}\sqrt{2}$

$\alpha = 1\frac{1}{4}\pi \quad \vee \quad \alpha = 1\frac{3}{4}\pi$

d. $\cos\alpha = 0$

$\alpha = \frac{1}{2}\pi \quad \vee \quad \alpha = 1\frac{1}{2}\pi$

e. $\cos\alpha = \frac{1}{2}\sqrt{3}$

$\alpha = \frac{1}{6}\pi \quad \vee \quad \alpha = 1\frac{5}{6}\pi$

f. $\cos\alpha = \frac{1}{2}\sqrt{2}$

$\alpha = \frac{1}{4}\pi \quad \vee \quad \alpha = 1\frac{3}{4}\pi$

Opgave 41:

$..., -2\tfrac{1}{2}\pi, -1\tfrac{1}{2}\pi, -\tfrac{1}{2}\pi, \tfrac{1}{2}\pi, 1\tfrac{1}{2}\pi, 2\tfrac{1}{2}\pi, ...$

Opgave 42:

a. $\sin(3x - \tfrac{1}{2}\pi) = 0$

$3x - \tfrac{1}{2}\pi = 0 + k \cdot \pi$

$3x = \tfrac{1}{2}\pi + k \cdot \pi$

$x = \tfrac{1}{6}\pi + k \cdot \tfrac{1}{3}\pi$

b. $\cos(\tfrac{1}{2}x - \tfrac{1}{6}\pi) = 0$

$\tfrac{1}{2}x - \tfrac{1}{6}\pi = \tfrac{1}{2}\pi + k \cdot \pi$

$\tfrac{1}{2}x = \tfrac{2}{3}\pi + k \cdot \pi$

$x = 1\tfrac{1}{3}\pi + k \cdot 2\pi$

c. $\sin^2 x = \sin x$

$\sin^2 x - \sin x = 0$

$\sin x(\sin x - 1) = 0$

$\sin x = 0 \quad \vee \quad \sin x = 1$

$x = 0 + k \cdot \pi \quad \vee \quad x = \tfrac{1}{2}\pi + k \cdot 2\pi$

d. $\cos^2 2x + \cos 2x = 0$

$\cos 2x(\cos 2x + 1) = 0$

$\cos 2x = 0 \quad \neg \quad \cos 2x = -1$

$2x = \tfrac{1}{2}\pi + k \cdot \pi \quad \vee \quad 2x = \pi + k \cdot 2\pi$

$x = \tfrac{1}{4}\pi + k \cdot \tfrac{1}{2}\pi \quad \vee \quad x = \tfrac{1}{2}\pi + k \cdot \pi$

Opgave 43:

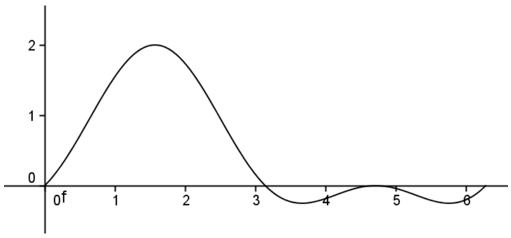
$\sin^2 x + \sin x = 0$

$\sin x(\sin x + 1) = 0$

$\sin x = 0 \quad \vee \quad \sin x = -1$

$x = 0 \quad \vee \quad x = \pi \quad \vee \quad x = 2\pi \quad \vee \quad x = 1\tfrac{1}{2}\pi$

$x = 0 \quad \vee \quad \pi \le x \le 2\pi$



Opgave 44:

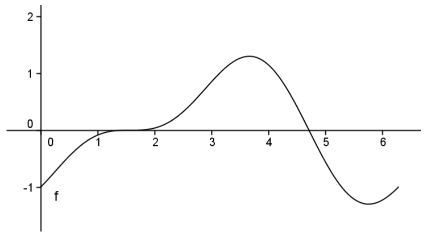
a. $\sin x \cdot \cos x - \cos x \le 0$

$\cos x(\sin x - 1) = 0$

$\cos x = 0 \quad \vee \quad \sin x = 1$

$x = \tfrac{1}{2}\pi \quad \vee \quad x = 1\tfrac{1}{2}\pi$

$0 \le x \le \tfrac{1}{2}\pi \quad \vee \quad 1\tfrac{1}{2}\pi \le x \le 2\pi$

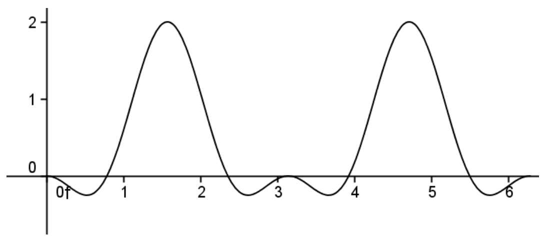


b. $\cos^2 2x - \cos 2x \ge 0$

$\cos 2x(\cos 2x - 1) = 0$

$\cos 2x = 0 \quad \vee \quad \cos 2x = 1$

$2x = \tfrac{1}{2}\pi + k \cdot \pi \quad \vee \quad 2x = 0 + k \cdot 2\pi$



$x = \tfrac{1}{4}\pi \quad \vee \quad x = \tfrac{3}{4}\pi \quad \vee \quad x = 1\tfrac{1}{4}\pi \quad \vee \quad x = 1\tfrac{3}{4}\pi \quad \vee \quad x = 0 \quad \vee \quad x = \pi \quad \vee \quad x = 2\pi$

$x = 0 \quad \vee \quad \tfrac{1}{4}\pi \le x \le \tfrac{3}{4}\pi \quad \vee \quad x = \pi \quad \vee \quad 1\tfrac{1}{4}\pi \le x \le 1\tfrac{3}{4}\pi \quad \vee \quad x = 2\pi$

Opgave 45:

a. $\sin\frac{1}{6}\pi = \frac{1}{2}$

b. $\sin 2\frac{1}{6}\pi = \sin\frac{1}{6}\pi = \frac{1}{2}$

$\sin 4\frac{1}{6}\pi = \sin\frac{1}{6}\pi = \frac{1}{2}$

c. $\sin\frac{5}{6}\pi = \frac{1}{2}$

d. $\sin 2\frac{5}{6}\pi = \sin\frac{5}{6}\pi = \frac{1}{2}$

$\sin(-1\frac{1}{6}\pi) = \sin\frac{5}{6}\pi = \frac{1}{2}$

Opgave 46:

a. $2\sin\frac{1}{2}x = 1$

$\sin\frac{1}{2}x = \frac{1}{2}$

$\frac{1}{2}x = \frac{1}{6}\pi + k\cdot 2\pi \quad \vee \quad \frac{1}{2}x = \frac{5}{6}\pi + k\cdot 2\pi$

$x = \frac{1}{3}\pi + k\cdot 4\pi \quad \vee \quad x = \frac{5}{3}\pi + k\cdot 4\pi$

b. $2\cos(x - \frac{1}{3}\pi) = 1$

$\cos(x - \frac{1}{3}\pi) = \frac{1}{2}$

$x - \frac{1}{3}\pi = \frac{1}{3}\pi + k\cdot 2\pi \quad \vee \quad x - \frac{1}{3}\pi = -\frac{1}{3}\pi + k\cdot 2\pi$

$x = \frac{2}{3}\pi + k\cdot 2\pi \quad \vee \quad x = 0 + k\cdot 2\pi$

c. $2\sin(2x - \frac{1}{4}\pi) = -\sqrt{3}$

$\sin(2x - \frac{1}{4}\pi) = -\frac{1}{2}\sqrt{3}$

$2x - \frac{1}{4}\pi = 1\frac{1}{3}\pi + k\cdot 2\pi \quad \vee \quad 2x - \frac{1}{4}\pi = 1\frac{2}{3}\pi + k\cdot 2\pi$

$2x = 1\frac{7}{12}\pi + k\cdot 2\pi \quad \vee \quad 2x = 1\frac{11}{12}\pi + k\cdot 2\pi$

$x = \frac{19}{24}\pi + k\cdot \pi \quad \vee \quad x = \frac{23}{24}\pi + k\cdot \pi$

d. $2\cos(3x - \pi) = -1$

$\cos(3x - \pi) = -\frac{1}{2}$

$3x - \pi = \frac{2}{3}\pi + k\cdot 2\pi \quad \vee \quad 3x - \pi = -\frac{2}{3}\pi + k\cdot 2\pi$

$3x = 1\frac{2}{3}\pi + k\cdot 2\pi \quad \vee \quad 3x = \frac{1}{3}\pi + k\cdot 2\pi$

$x = \frac{5}{9}\pi + k\cdot \frac{2}{3}\pi \quad \vee \quad x = \frac{1}{9}\pi + k\cdot \frac{2}{3}\pi$

Opgave 47:

a. $2\sin(2x - \frac{1}{6}\pi) = \sqrt{2}$

$\sin(2x - \frac{1}{6}\pi) = \frac{1}{2}\sqrt{2}$

$2x - \frac{1}{6}\pi = \frac{1}{4}\pi + k\cdot 2\pi \quad \vee \quad 2x - \frac{1}{6}\pi = \frac{3}{4}\pi + k\cdot 2\pi$

$2x = \frac{5}{12}\pi + k\cdot 2\pi \quad \vee \quad 2x = \frac{11}{12}\pi + k\cdot 2\pi$

$x = \frac{5}{24}\pi + k\cdot \pi \quad \vee \quad x = \frac{11}{24}\pi + k\cdot \pi$

$x = \frac{5}{24}\pi \quad \vee \quad x = \frac{11}{24}\pi \quad \vee \quad x = 1\frac{5}{24}\pi \quad \vee \quad x = 1\frac{11}{24}\pi$

b. $2\cos(3x - \frac{1}{2}\pi) = \sqrt{3}$

$\cos(3x - \frac{1}{2}\pi) = \frac{1}{2}\sqrt{3}$

$3x - \frac{1}{2}\pi = \frac{1}{6}\pi + k\cdot 2\pi \quad \vee \quad 3x - \frac{1}{2}\pi = -\frac{1}{6}\pi + k\cdot 2\pi$

$3x = \frac{2}{3}\pi + k\cdot 2\pi \quad \vee \quad 3x = \frac{1}{3}\pi + k\cdot 2\pi$

$x = \frac{2}{9}\pi + k \cdot \frac{2}{3}\pi \quad \vee \quad x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$

$x = \frac{1}{9}\pi \quad \vee \quad x = \frac{2}{9}\pi \quad \vee \quad x = \frac{7}{9}\pi \quad \vee \quad x = \frac{8}{9}\pi \quad \vee \quad x = 1\frac{4}{9}\pi \quad \vee \quad x = 1\frac{5}{9}\pi$

c. $\sin \frac{2}{3}x = -\frac{1}{2}\sqrt{2}$

$\frac{2}{3}x = 1\frac{1}{4}\pi + k \cdot 2\pi \quad \vee \quad \frac{2}{3}x = 1\frac{3}{4}\pi + k \cdot 2\pi$

$x = 1\frac{7}{8}\pi + k \cdot 3\pi \quad \vee \quad x = 2\frac{5}{8}\pi + k \cdot 3\pi$

$x = 1\frac{7}{8}\pi$

d. $\cos \frac{1}{2}x = -\frac{1}{2}\sqrt{3}$

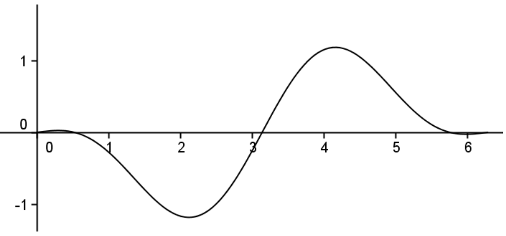
$\frac{1}{2}x = \frac{2}{3}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}x = \frac{4}{3}\pi + k \cdot 2\pi$

$x = 1\frac{1}{3}\pi + k \cdot 4\pi \quad \vee \quad x = 2\frac{2}{3}\pi + k \cdot 4\pi$

$x = 1\frac{1}{3}\pi$

Opgave 48:

a. $\sin x(\cos x - \frac{1}{2}\sqrt{3}) \geq 0$

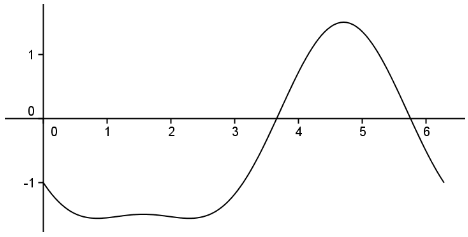


$\sin x = 0 \quad \vee \quad \cos x = \frac{1}{2}\sqrt{3}$

$x = 0 \quad \vee \quad x = \pi \quad \vee \quad x = 2\pi \quad \vee \quad x = \frac{1}{6}\pi \quad \vee \quad x = 1\frac{5}{6}\pi$

$0 \leq x \leq \frac{1}{6}\pi \quad \vee \quad \pi \leq x \leq 1\frac{5}{6}\pi \quad \vee \quad x = 2\pi$

b. $\sin^2 x - 1\frac{1}{2}\sin x - 1 \leq 0$



$p^2 - 1\frac{1}{2}p - 1 = 0$

$(p-2)(p+\frac{1}{2}) = 0$

$p = 2 \quad \vee \quad p = -\frac{1}{2}$

$\sin x = 2 \quad \vee \quad \sin x = -\frac{1}{2}$

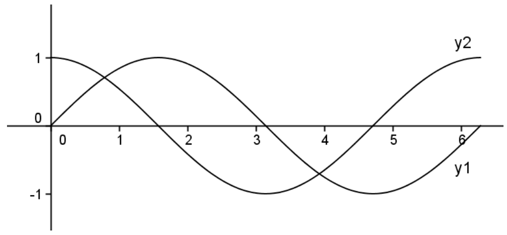
k.n. $\qquad x = 1\frac{1}{6}\pi \quad \vee \quad x = 1\frac{5}{6}\pi$

$0 \leq x \leq 1\frac{1}{6}\pi \quad \vee \quad 1\frac{5}{6}\pi \leq x \leq 2\pi$

12.5 Goniometrische functies differentiëren.

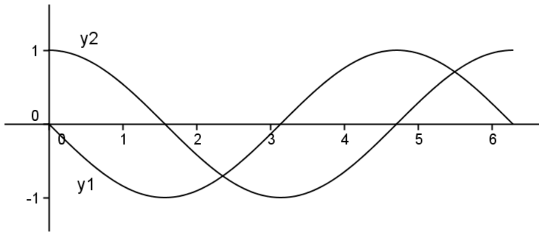
Opgave 49:

a.



b. $y = \cos x$

c.



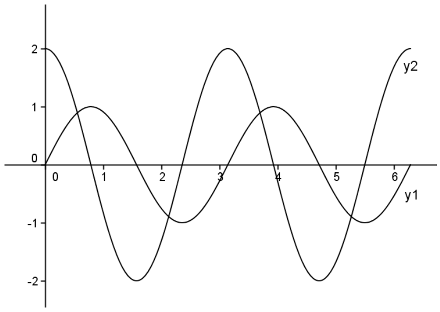
$y = -\sin x$

d. $y = \sin(x-2)$ $\qquad$ $y = \cos(x+1)$

$y' = \cos(x-2)$ $\qquad$ $y' = -\sin(x+1)$

Opgave 50:

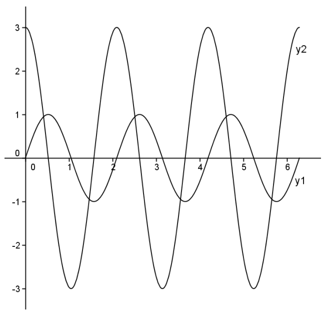
a.



$y_1: \quad per = \pi \quad amp = 1$

$y_2: \quad per = \pi \quad amp = 2$

b.



$y_1: \quad per = \frac{2}{3}\pi \quad amp = 1$

$y_2: \quad per = \frac{2}{3}\pi \quad amp = 3$

c. $a = 2: \quad y_2 = 2\cos 2x$

$a = 3: \quad y_2 = 3\cos 3x$

Opgave 51:
$y = \cos 3x$
$y' = -3\sin 3x$

Opgave 52:
a. $f(x) = \cos 2x = \cos u$ met $u = 2x$
dus $u' = 2$
$f'(x) = -\sin u \cdot u' = -\sin 2x \cdot 2 = -2\sin 2x$
b. $g(x) = x \cdot \cos x$
$g'(x) = 1 \cdot \cos x + x \cdot -\sin x = \cos x - x \cdot \sin x$
c. $h(x) = 3 + 4\sin(2x - \frac{1}{3}\pi) = 3 + 4\sin u$ met $u = 2x - \frac{1}{3}\pi$
dus $u' = 2$
$h'(x) = 4\cos u \cdot u' = 4\cos(2x - \frac{1}{3}\pi) \cdot 2 = 8\cos(2x - \frac{1}{3}\pi)$
d. $j(x) = 10 + 16\sin(\frac{1}{2}(x-1)) = 10 + 16\sin u$ met $u = \frac{1}{2}(x-1)$
dus $u' = \frac{1}{2}$
$j'(x) = 16\cos u \cdot u' = 16\cos(\frac{1}{2}(x-1)) \cdot \frac{1}{2} = 8\cos(\frac{1}{2}(x-1))$

Opgave 53:
a. $f(x) = \sin(ax+b) = \sin u$ met $u = ax + b$
dus $u' = a$
$f'(x) = \cos u \cdot u' = \cos(ax+b) \cdot a = a \cdot \cos(ax+b)$
b. $g(x) = \cos(ax+b) = \cos u$ met $u = ax + b$
dus $u' = a$
$g'(x) = -\sin u \cdot u' = -\sin(ax+b) \cdot a = -a \cdot \sin(ax+b)$

Opgave 54:
a. $f(x) = x \cdot \sin 2x$
$f'(x) = 1 \cdot \sin 2x + x \cdot \cos 2x \cdot 2 = \sin 2x + 2x \cdot \cos x$
b. *

Opgave 55:
a. $f(x) = x \cdot \cos 2x$
$f'(x) = 1 \cdot \cos 2x + x \cdot -\sin 2x \cdot 2 = \cos 2x - 2x \cdot \sin 2x$
b. $g(x) = x^2 \cdot \sin 3x$
$g'(x) = 2x \cdot \sin 3x + x^2 \cdot \cos 3x \cdot 3 = 2x \cdot \sin 3x + 3x^2 \cdot \cos 3x$
c. $h(x) = 2x \cdot \sin(3x-1)$
$h'(x) = 2 \cdot \sin(3x-1) + 2x \cdot \cos(3x-1) \cdot 3 = 2\sin(3x-1) + 6x \cdot \cos(3x-1)$
d. $j(x) = 1 + 3x \cdot \cos \frac{1}{2}x$
$j'(x) = 3 \cdot \cos \frac{1}{2}x + 3x \cdot -\sin \frac{1}{2}x \cdot \frac{1}{2} = 3\cos \frac{1}{2}x - 1\frac{1}{2}x \cdot \sin \frac{1}{2}x$

Opgave 56:
a. I: $f(x) = \sin^2 x = \sin x \cdot \sin x$
$f'(x) = \cos x \cdot \sin x + \sin x \cdot \cos x = 2\sin x \cos x$

II: $f(x) = \sin^2 x = u^2$ met $u = \sin x$

dus $u' = \cos x$

$f'(x) = 2u \cdot u' = 2\sin x \cdot \cos x$

b. *

Opgave 57:

a. $f(x) = \cos^2 x = u^2$ met $u = \cos x$

dus $u' = -\sin x$

$f'(x) = 2u \cdot u' = 2\cos x \cdot -\sin x = -2\cos x \sin x$

b. $g(x) = 2\sin^2 x = 2u^2$ met $u = \sin x$

dus $u' = \cos x$

$g'(x) = 4u \cdot u' = 4\sin x \cdot \cos x$

c. $h(x) = 1 + 2\cos^2 x = 1 + 2u^2$ met $u = \cos x$

dus $u' = -\sin x$

$h'(x) = 4u \cdot u' = 4\cos x \cdot -\sin x = -4\cos x \sin x$

d. $j(x) = x + 3\sin^2 x = x + 3u^2$ met $u = \sin x$

dus $u' = \cos x$

$j'(x) = 1 + 6u \cdot u' = 1 + 6\sin x \cdot \cos x$

Opgave 58:

a. $f(x) = \sin^3 x = u^3$ met $u = \sin x$

dus $u' = \cos x$

$f'(x) = 3u^2 \cdot u' = 3\sin^2 x \cdot \cos x$

b. $g(x) = x \cdot \sin^2 x$

$g'(x) = 1 \cdot \sin^2 x + x \cdot 2\sin x \cos x = \sin^2 x + 2x \sin x \cos x$

c. $h(x) = \sqrt{2 + \sin x} = \sqrt{u}$ met $u = 2 + \sin x$

dus $u' = \cos x$

$$h'(x) = \frac{1}{2\sqrt{u}} \cdot u' = \frac{1}{2\sqrt{2 + \sin x}} \cdot \cos x = \frac{\cos x}{2\sqrt{2 + \sin x}}$$

d. $j(x) = 2x \cdot \cos(x^2)$

$j'(x) = 2 \cdot \cos(x^2) + 2x \cdot -\cos(x^2) \cdot 2x = 2\cos(x^2) - 4x^2 \sin(x^2)$

Opgave 59:

a. $f(x) = \cos^2 x - \cos x$

$f'(x) = 2\cos x \cdot -\sin x + \sin x$

$= -2\cos x \sin x + \sin x = 0$

$\sin x(-2\cos x + 1) = 0$

$\sin x = 0 \quad \vee \quad -2\cos x = -1$

$\sin x = 0 \quad \vee \quad \cos x = \frac{1}{2}$

$x = 0 \quad \vee \quad x = \pi \quad \vee \quad x = 2\pi \quad \vee \quad x = \frac{1}{3}\pi \quad \vee \quad x = 1\frac{2}{3}\pi$